A risk-reward balancing model of generation side and purchasing side based on optimized total risk-reward

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Abstract

This paper used the Modern Portfolio Theory (MPT) to analyze the risk-reward of purchasing side and generation side by purchasing-selling power from multi-market, considered the game between the risk-reward of purchasing side and generation side and built a balancing risk-reward model of two sides in the condition of the optimal total risk-reward of both sides. Based on the total optimal risk-reward, the model optimized the purchasing-selling power proportion portfolio to make the risk-reward of both sides in their acceptable range and increase their reward at the same time. The simulation example confirmed the validity and suitability of the model, and manifested that the proposed model provides both the power purchasing side and the generation side some reference and guiding value for purchasing decision-making.

Keywords: MPT, electricity market, power purchase portfolio, power selling portfolio, balancing risk-reward

1 Introduction

With the electricity market reform, the purchasing side and generation side formed a multiple purchasing-selling power markets in domestic, mainly including the contract market, the spot market as well as ancillary services markets, and the options markets and futures markets may form in the future [1]. Due to the different operating mechanism and delivery time, the purchasing-selling power markets have different costs and price of trading electricity and the fluctuations in costs and prices are also different. In this context, the transactions proportion in different markets of the purchase side and generation side will lead to different risk-reward [2-3]. The appearance of multi-market in purchasing side and generation side makes it possible that using the financial theory and techniques to electricity market, and help to make the rational decisions of purchasing-selling power in both purchasing side and generation side [4-8].

The common goal of purchasing side and generation side is the total risk profit maximization in both sides. In this goal, there is a reward game between the purchase side and generation side [9-10], which is to say that one side increase will inevitably lead decrease in another side. So in the ideal state, there is a dynamic equilibrium point between the interests of both sides. Literature [2-10] used the mean-variance method, VAR method, CVAR method to analyze the purchase behavior on the spot market, contract market, futures market and get the optimal ratio so that the risk-reward of the single side is optimal. Most of these literatures are only considering the maximization reward of purchasing side or generating side and ignoring the effect from the constraints and game results between two sides. In practice, the proportions on each market of purchasing side and selling side should consider the risk-reward of both sides. Based on the optimal total risk-reward in purchasing side and generating side, this paper concluded the risk-reward balance point through the analysis of the risk-reward of different market portfolio. The equilibrium point meets the conditions that the risk of two sides is within reasonable limits as well as the maximize reward.

2 The benefits analysis of purchasing side and generating side

For any one market, the income of selling electricity on generating side depends on the two parts, one is the cost of electricity p_1, the other is tariff p_2; the income on purchase side also depends on two parts, one is tariff p_2, the other is sales price p_3. The benefit of generating side is expressed as $r = p_2 - p_1$; the benefit of purchasing side is expressed as $r' = p_3 - p_2$; the total risk-reward is expressed as $R = p_3 - p_1$.

For the different markets, the prices are changing randomly, so the benefits of two sides in each market are also random. We can use the expectations to represent the benefits, and use its variance to represent the risk. Assuming that the two sides purchase and sale electricity in T market portfolio which including I markets, and according to the earnings variance method, the risk-reward of generating side can be expressed as:

$$E(r_T) = \sum_{i=1}^{I} \alpha_i E(r_i) , \tag{1}$$

$$\sigma_T^2 = \sum_{i=1}^{I} \sum_{j=1}^{I} \alpha_i \alpha_j COV(r_i, r_j) . \tag{2}$$

The risk-reward of purchasing side is:

$$E(r_T') = \sum_{i=1}^{I} \alpha_i E(r_i') , \tag{3}$$

$$\sigma_T'^2 = \sum_{i=1}^{I}\sum_{j=1}^{I}\alpha_i\alpha_j COV(r_i', r_j') . \quad (4)$$

The total risk-reward has the same express methods.

Among them, $E(r_T)$, $E(r_T')$ means the income of generation side and purchase side. σ_T^2, σ'^2 means the risk of generation side and purchase side. α_i , α_j represents the purchasing or selling electricity proportion of the first i , j market. r_i , r_j means the income of the first i , market in sale side. r_i' , r_j' means the income of the first i , j market in purchasing side. $COV(r_i, r_j)$, $COV(r_i', r_j')$ means the covariance of income in the first i , j market in the two sides.

3 Total risk-reward optimization models

Because of the common goals that total risk-reward optimization in purchase side and selling side, the total risk-reward optimization problem is a two-objective optimization problem, which not only hope to gain the most, but also minimize the risks. The optimization problem can be expressed as follows:

$$\begin{cases} \max E(R_T) = \sum_{i=1}^{I}\alpha_i E(R_i) \\ \min \sigma_T^2 = \sum_{i=1}^{I}\sum_{j=1}^{I}\alpha_i\alpha_j COV(R_i, R_j) \\ st. \sum_{i=1}^{I}\alpha_i = 1 \\ \alpha_i \ge 0, i = 1, 2, ..., I \end{cases} \quad (5)$$

Among this, R_i , R_j represent the total benefits in the first i , j market.

The above question can be transformed for the single objective optimization question of reward under the certain risk, namely:

$$\begin{cases} \max E(R_T) = \sum_{i=1}^{I}\alpha_i E(R_i) \\ st. \sum_{i=1}^{I}\alpha_i = 1 \\ \quad \alpha_i \ge 0, i = 1, 2, ..., I \\ \quad \min \sigma_T^2 = \sigma^2 \end{cases} \quad (6)$$

We can know that, this optimization problem can obtain a solution set, which means the biggest reward purchase portfolio under different risk status. The image is the upper part of a hyperbolic, called the efficient frontier of the market portfolio problem.

4 Reward equilibrium models of generation side and purchase side under the optimal total risk-reward

In optimal total risk-reward conditions, due to the existence of the game between purchase side and generation side, both sides have sought to maximum risk-reward. And these two issues may be negatively correlated, so the risk-reward of generation side and purchase side is a dynamic equilibrium problem, which depending on the game forces on both sides. It is the multi-objective problems that seeking the optimal total risk-reward and the optimal risk-reward of each purchase side and generation side. Among them, total risk-reward is the most important goal, and then tries to reach the effective border in the two sides. In general, there is not an optimal solution meeting the three targets.

This can be divided into two cases: 1) If the optimal solution exists, use the optimal solution of total risk-reward to search the solution in the objective function of purchase side and selling side respectively (refer to the total risk optimal model) and meet the requirements of bilateral efficient frontier. 2) If the optimal solution of the optimization problem does not exist, it is indicated that the two sides cannot reach an optimal solution at the same time due to the constraints from each other. So we can only seeking for the suboptimal one, which allows errors between optimal market portfolio and bilateral efficient frontier. The complete search process is shown in Figure 1.

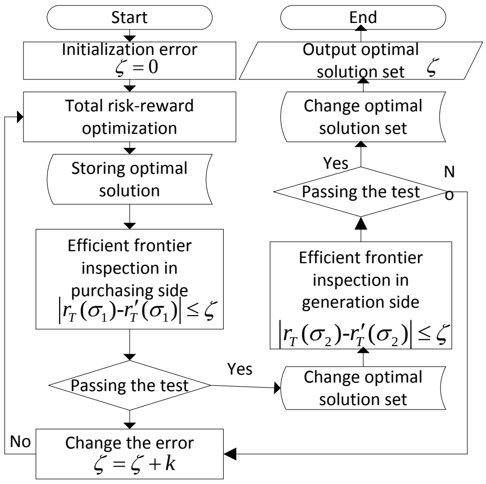

FIGURE 1 Optimization search process

$|r_T(\sigma_1) - r_T'(\sigma_1)|$ is for the difference between efficient frontier reward of purchase side and the actual reward in the same risk status. $|r_T(\sigma_2) - r_T'(\sigma_2)|$ is for the difference between efficient frontier reward of generation side and the actual reward in the same risk status. ζ is for the difference allowed.

The risk balanced of purchase side and generation side do not represented equal risk, but their risks are in their acceptable range. Purchasing-selling power proportion portfolio of purchase side and generation side need to satisfy the own indifference curves law, and reduce the efficient frontier to an acceptable segment according to their respective indifference curves. Then we should search in the acceptable segments.

5 Example simulation

5.1 CONDITION ASSUMPTION

There are long-term contract market, mid-term contract market and spot market in power purchasing market of one region, the power purchase ratio are 0.3,0.6,0.1. The sale price of the region is stable relatively, for 650 yuan / MWh. The generation costs is influenced by coal prices, which is a random variable. The tariff is also a random variable effected by market characteristics and costs. According to historical data, we can calculate the income generation side, the income of purchasing side and total income as shown in Table 1.

TABLE 1 The basic situation of purchase side and generation side in one region

Market		**Spot market**	**Mid-term contract**	**Long-term contract**
Reward	Generation side	258.500	254.667	248.167
	Purchase side	208.667	207.667	202.833
	Total reward	467.167	462.333	451.000
Risk(standard deviation)	Generation side	40.722	20.539	11.531
	Purchase side	40.332	12.801	12.024
	Total reward	29.789	9.070	5.865

According to the historical data, the covariance of generation side, the purchase side and total reward in each market are as follows:

$$U_1=\begin{bmatrix}1658.30 & -794.40 & 97.10\\ -794.40 & 421.87 & -60.93\\ 97.10 & -60.93 & 132.97\end{bmatrix},$$

$$U_2=\begin{bmatrix}1626.67 & -254.93 & 340.53\\ -254.93 & 163.87 & -37.27\\ 340.53 & -37.27 & 132.97\end{bmatrix},$$

$$U_3=\begin{bmatrix}887.37 & -125.27 & 65.00\\ -125.27 & 82.27 & -2.80\\ 65.00 & -2.80 & 34.40\end{bmatrix}.$$

5.2 RISK BALANCED OPTIMIZATION BASED ON OPTIMAL TOTAL REWARD

The comparison of total reward, generation side and purchase side efficient frontier are shown in Figure 2 and Figure 3.

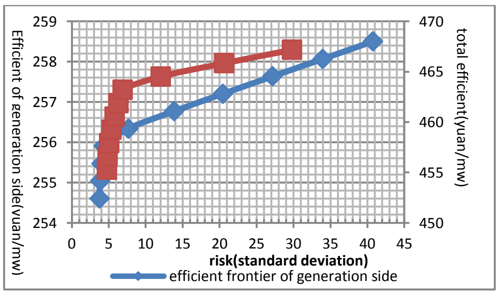

FIGURE 2 Efficient frontier of Total reward and purchase side

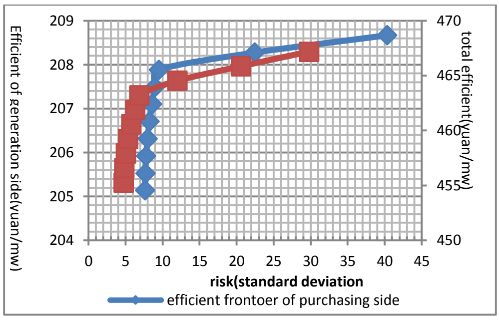

FIGURE 3 Efficient frontier of Total reward and generation side

From Figure 2 and Figure 3, at the beginning, the slope of efficient frontier in purchase side and generation side are large and with clear growth trend. Then the curve maintains a certain stage and the upward trend is slowing down. In most cases, both purchase side and generation side do not want to accept the ends of efficient frontier. Because the front-end have a minimal risk but with a small reward, and the back-end have a maximum reward with far more risks.

Based on the indifference curves of risk-reward in generation side and purchase side, we can determine an acceptable segment of efficient frontier which have a larger efficient frontier reward and total risk-reward. The acceptable segment of efficient frontier in generation side of this area is $4 \le \sigma_1 \le 12$. The acceptable segment of efficient frontier in purchasing side is $8 \le \sigma_2 \le 11$.

Since the solution of total reward on the efficient frontier is an infinite solution set, here we take 100 equally points on the efficient frontier of the total reward as its efficient frontier solution set.

When ζ=0.25 , after the optimized loop, the optimal solution set are shown in Figure 4 and Figure 5. At this point, the purchase side has the optimal solution set, while the generation side has no optimal solution set.

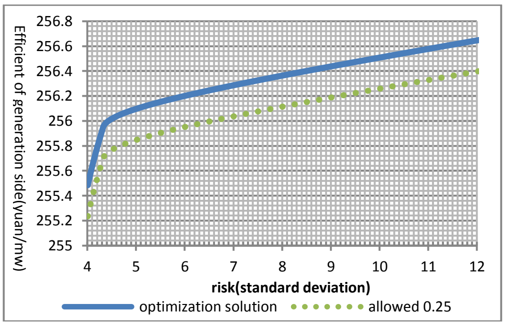

FIGURE 4 ζ=0.25 efficient frontier test in purchasing side

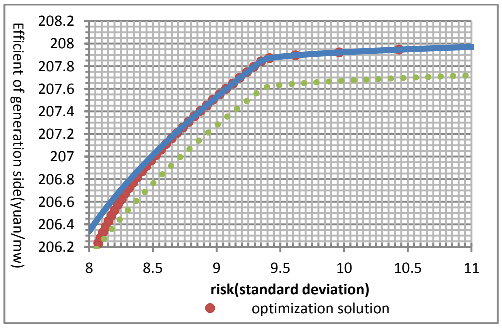

FIGURE 5 ζ=0.25 efficient frontier test in generation side

When ζ=0.5 , after the optimized loop, the optimal solution set are shown in Figure 6 and Figure7. At this point, the purchase side has the optimal solution set, and there is the optimal solution meeting the efficient frontier test in generation side.

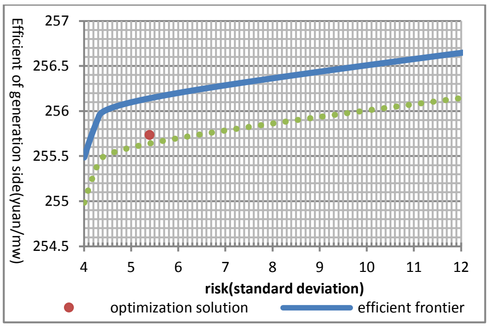

FIGURE 6 ζ=0.5 efficient frontier test in purchasing side

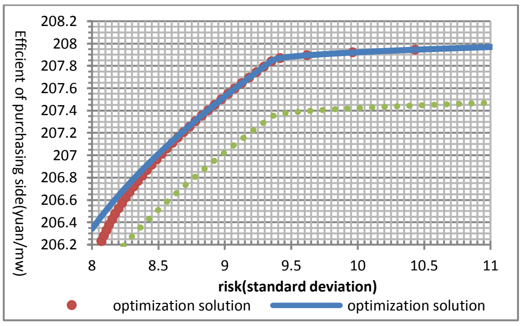

FIGURE 7 ζ=0.5, efficient frontier test in generation side

At this case, the optimal solution is shown in Table 2.

TABLE 2 Optimal solution

Purchase Market	Spot market	Mid-term contract	Long-term contract
Optimal ratio	0.2781	0.7219	0

5.3 RESULT ANALYSIS

The result analysis is shown as Table 3. After optimization, the reward of regional generation side, purchase side and total reward increased. As for a big city, there will be a huge increasing when total reward increased 1 yuan every Mw.h. This optimization result makes the risk of generation side increase slightly and leads to the reward decrease in total reward and purchase side. In order to reduce the systemic risk, both sides can make negotiation with each other to decide subsidies. The optimization results that do not purchase power from long-term contracts market is because of risk range. In this market, mid-term contracts have already reduced the risk to a considerable range, while the long-term contracts market is so stable that leading to reward loss.

TABLE3 The results before and after optimization

Risk-reward / Power purchase ratio			Total risk-reward		Risk-reward in generation side		Risk-reward in purchasing side	
			Reward (yuan /mw.h)	Risk (standard deviation)	Reward (yuan/mw.h)	Risk (standard deviation)	Reward (yuan/mw.h)	Risk (standard deviation)
The original power purchase proportion	X1	0.3	462.6500	8.2638	255.1667	3.8702	207.4833	11.4465
	X2	0.6						
	X3	0.1						
Optimal power purchase proportion	X1	0.2781	463.6777	7.8241	255.733	5.396	207.945	10.434
	X2	0.7219						
	X3	0						
Optimization effect	X1	decline	Increase 1.0277yuan/MW .h	Declined a little	Increase 0.5663yuan/MW .h	Increased a little	Increase 0.4617yuan/MW .h	Declined a little
	X2	increase						
	X3	No purchase						

6 Conclusions

Due to the cost and price of different purchasing-selling power markets, the power purchase proportions in different markets have a direct effect to the risk-reward of purchasing-selling both sides. The common goal of purchasing side and generation side is the total risk profit maximization in both sides. In this goal, there is a dynamic balance between the purchase side and generation side. This article provides the optimization model based on the maximum total risk-reward of purchase side and generation side, and get a balanced optimal power purchase proportion. The proportion makes the risk-reward of two sides stay in their acceptable range, and ensures that the risk-reward of each side close to their efficient frontier when the total reward reached the maximum.

It should be noted that the case studies tell us that although the optimization results can make the reward of two sides increase, the results are not entirely balanced. It is possible to favor one side (the optimization result in this case tends to purchase side), so that increasing the systemic risk. Then we need a subsidy mechanism to neutralize the systemic risk. How to create the mechanism to minimum the total system risk is the next research content.

Acknowledgements

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